# Finite Density QCD at Strong Coupling

Alejandro Vaquero
University of Zaragoza

# Overview

1. Motivation
2. Lattice QCD and the Chemical Potential:
   The Sign Problem
3. The MDP proposal
4. Modifications of the MDP original algorithm
5. Results

# Motivation

- Astrophysics:
  Early universe, dense objects...
- Heavy ion collisions (LHC...)
- Strong coupling gives the right physics with the wrong numbers

# The Sign Problem

- Monte-Carlo simulations
  - Partition function $Z = \int e^{-S_E[q]} Dq =$

$$\int DU\, d\psi\, d\overline{\psi}\, e^{-\overline{\psi}\Delta\psi} e^{-S_G} = \int DU\, e^{-S_G}\, Det[\Delta(U)]$$

  - Action

$$S_E = S_G + S_F \quad \begin{cases} S_G = -Tr\left(\frac{\beta}{6}\sum_{\{\mu\nu\}} U_{\mu\nu}\right) \\ S_F = \overline{\psi}\Delta\psi \end{cases}$$

  - Fermionic matrix

$$\Delta = 2m + \eta_\mu(x)\left[U_\mu(x) - U_\mu^+(x + a\hat{e}_\mu)\right]$$

  - Boundary conditions
    - Periodic in X, Y and Z
    - Antiperiodic in T

# The Sign Problem

- At $\mu = 0$, $Det[\Delta(U)]$ is always positive

$$Z = \int DU e^{-S_G} Det[\Delta(U)]$$

- But if $\mu \neq 0$, $Det[\Delta(U)]$ may become negative (and it does!!)
- Naive solution: Introduce the sign in the observable $O$

$$\langle O \rangle = \frac{\int DU O(U, \overline{\psi}, \psi, \mu) e^{-S_G} Det[\Delta(U)]}{\int DU e^{-S_G} Det[\Delta(U)]}$$

- $e^{-S_E}$ can not be understood as a probability any more
  - Importance sampling not appliable!!!

# The Sign Problem

- Observables fluctuate wildly
- Exponentially increasing statistics required, as importance sampling does not apply
- The (in)famous Sign problem appears

# The MDP proposal

- Alternative strategy

$$Z = \int DU\, d\psi\, d\overline{\psi}\, e^{-\overline{\psi}\Delta\psi} e^{-S_G} = \int DU\, e^{-S_G} Det[\Delta(U)]$$

- Perform first gauge integration (Rossi and Wolff)
  - Is it possible?? YES, at strong coupling

$$S_G = -Tr\left(\frac{\beta}{6}\sum_{\{\mu\nu\}} U_{\mu\nu}\right) \xrightarrow{Strong\quad coupling} \beta \to 0$$

- The action becomes

$$Z = \int DU\, d\psi\, d\overline{\psi}\, e^{-\overline{\psi}(2m+i\Lambda)\psi}$$

- with

$$i\Lambda = \eta_j(x)\left[U_j(x) - U_j^+(x+a\hat{e}_j)\right] + \eta_4(x)\left[e^{\mu a}U_4(x) - e^{-\mu a}U_4^+(x+a\hat{e}_4)\right]$$

# The MDP proposal

- Gauge integration

$$Z = \int DU \int d\psi d\overline{\psi}\ e^{-\overline{\psi}\Delta\psi} = \int d\psi d\overline{\psi}\ e^{-2maM(x)} \int DU\ e^{-i\overline{\psi}\Lambda\psi}$$

- Defining

Monomer operator

$$M(x) = \sum_{a=1}^{3} \overline{\psi}^{a}(x)\psi^{a}(x)$$

Dimer operator

$$D_k(x,y) = \frac{1}{(k!)^2}\left[M(x)M(y)\right]^k$$

Barion–Antibarion fields

$$B(x) = \psi^1(x)\psi^2(x)\psi^3(x) \qquad \overline{B}(x) = \overline{\psi}^1(x)\overline{\psi}^2(x)\overline{\psi}^3(x)$$

# The MDP proposal

- Gauge integration

$$\int DU\, e^{-i\overline{\psi}\Lambda\psi} = 1 + \frac{1}{3} D_1(x,y) + \frac{1}{3} D_2(x,y) + D_3(x,y) + \zeta^3(x,y)\overline{B}(x)B(y) + \zeta^3(y,x)\overline{B}(y)B(x)$$

were

$$\zeta(x,y) = \eta_\lambda(x) \begin{cases} e^{\mu a} & if\ \lambda = 4 \\ -e^{-\mu a} & if\ \lambda = -4 \\ \pm 1 & if\ \lambda = \pm 1, \pm 2, \pm 3 \end{cases} \qquad y = x + a\hat{e}_\lambda$$

Dimers come from $U_\mu(x) U_\mu^+(x) = I$ cancellations

Barionic loops come from $\int DU [U_\mu(x)]^3 = I$

# The MDP proposal

- Mass term $$e^{2maM(x)} = 1 + 2ma(M(x)) + 4m^2a^2\frac{(M(x))^2}{2!} + 8m^3a^3\frac{(M(x))^3}{3!}$$

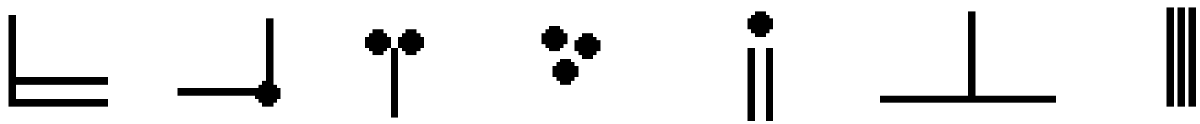
| Monomers | Dimers | Barionic loops |
|---|---|---|
| Live on sites | Live on links | Self avoiding |
| Mass term | Gauge terms | Weight depends on $\mu$ |
| Weight 2ma | Weight depends on $n_D$ | Large weight $e^{\pm 3kL_T\mu}$ |

└Up to three per site┘

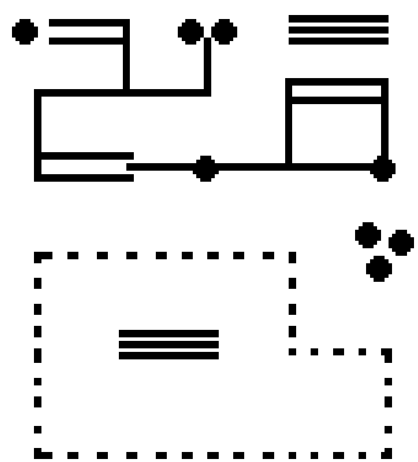
- Grassman integration rule $\longrightarrow$ $n_D + n_M = 3$

# The MDP proposal

- Possible vertices



- A configuration example



# The MDP proposal

- The sign problem is still there (regardless of the value of the chemical potential)

Sign of a barionic loop $\longrightarrow$ Depends only on its geometry

- Wilson loop

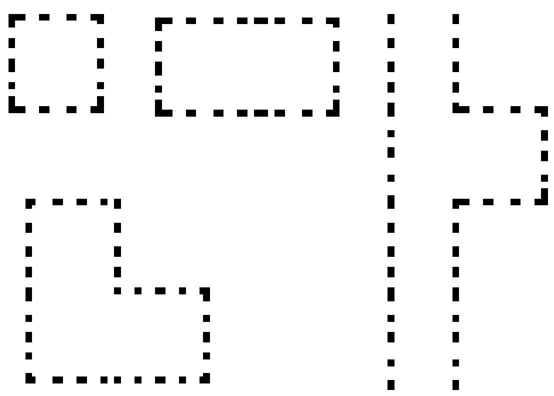
$$\sigma(C) = \prod_{\langle x,\lambda \rangle \in C} -(-1)^{N_-(C)} \eta_\lambda(x)$$

- Polyakov loop

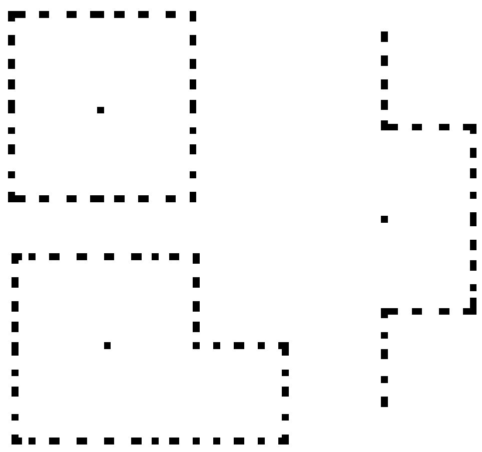
$$\sigma(C_k) = \prod_{\langle x,\lambda \rangle \in C_k} -(-1)^{N_-(C_k)+k(C_k)} \eta_\lambda(x)$$

# The MDP proposal

- Some example loops and their signs



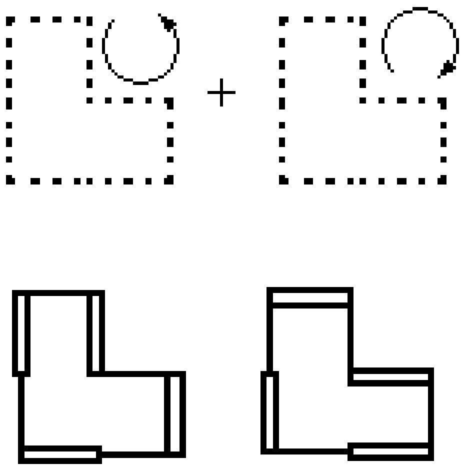
Positive loops

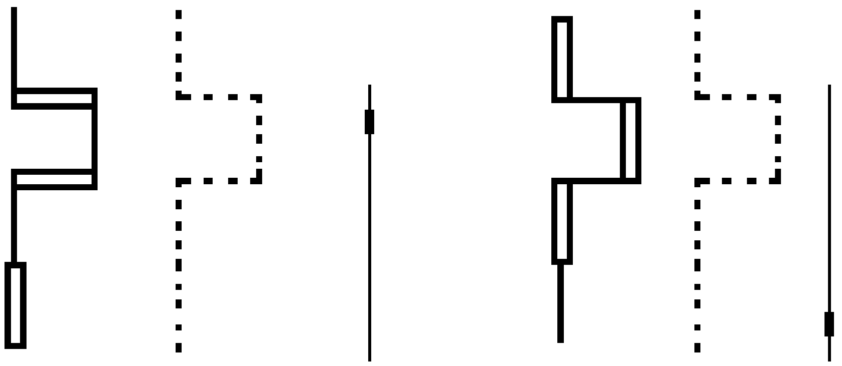


Negative loops

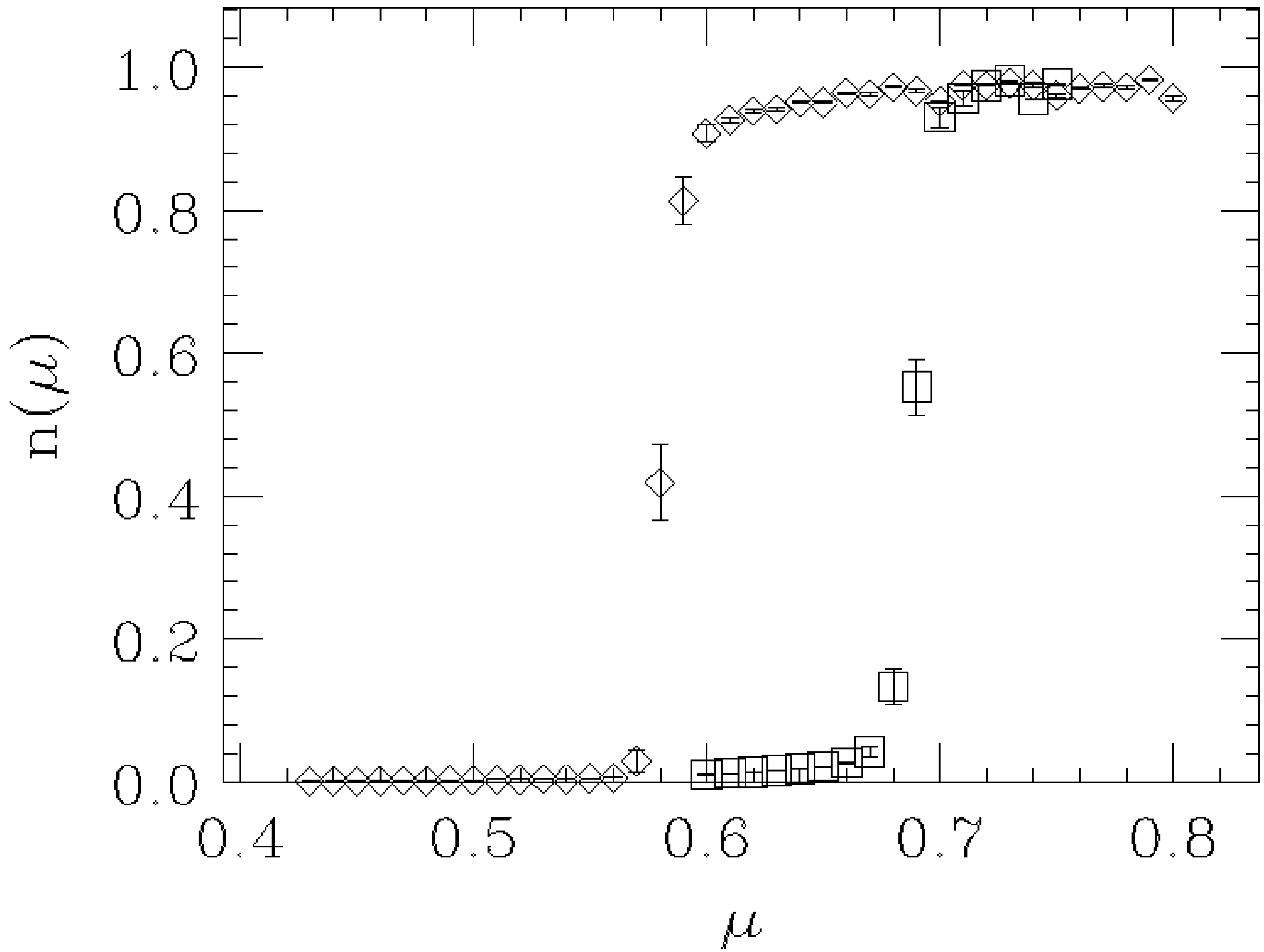
# The MDP proposal

- It can be solved for $\mu = 0$ by clustering configurations... (Karsch and Mütter)



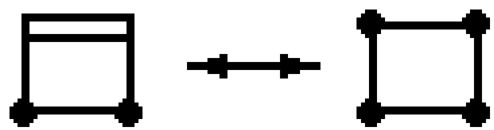



# The MDP proposal: $4^3$x4 and m = 0.1



# The MDP proposal

- Original simulation algorithm by Karsch and Mütter
  - Propose only one kind of change



  - Probability $\circ(m^2)$
  - As $m \to 0$ the acceptance drops
  - Barionic loops become extremely long-lived and non-interacting

**Ergodicity is lost (Azcoiti)**

  - Alternatives? Worm algorithms?? (Fromm and Forcrand)

# Modifications of the MDP original algorithm

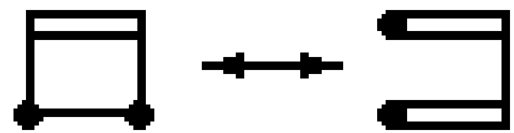
- Solution
  - Propose more changes

    Possibility of merging/splitting loops.

    Probability $\circ(1)$

    Possibility of displacing monomers.

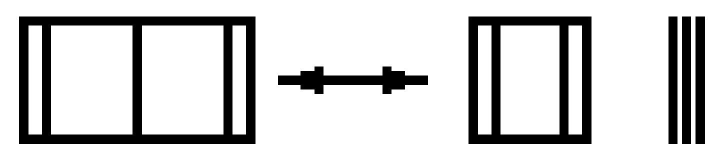
    Probability $\circ(1)$



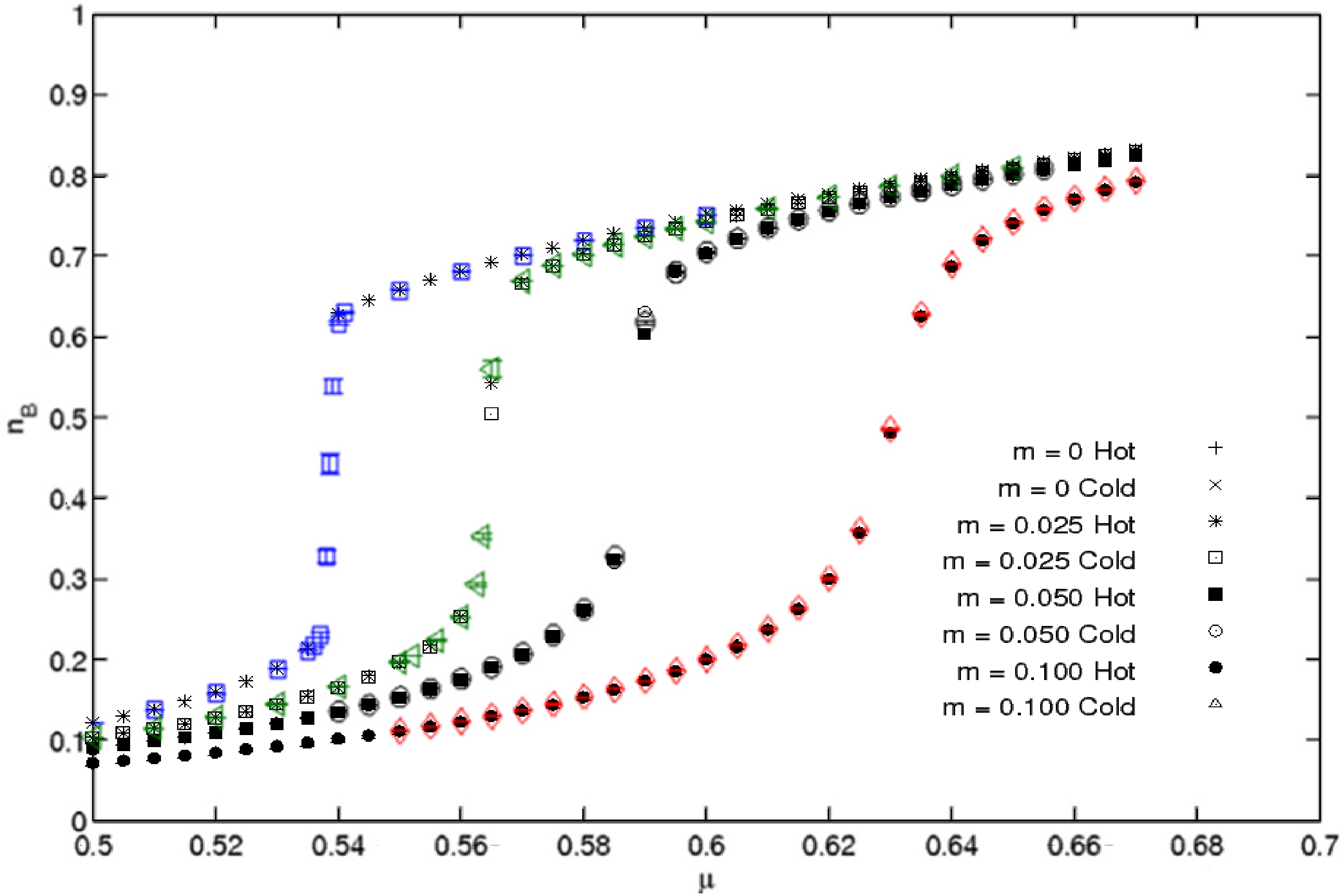
    Possibility of displacing dimers.

    Equivalent to a "local worm"

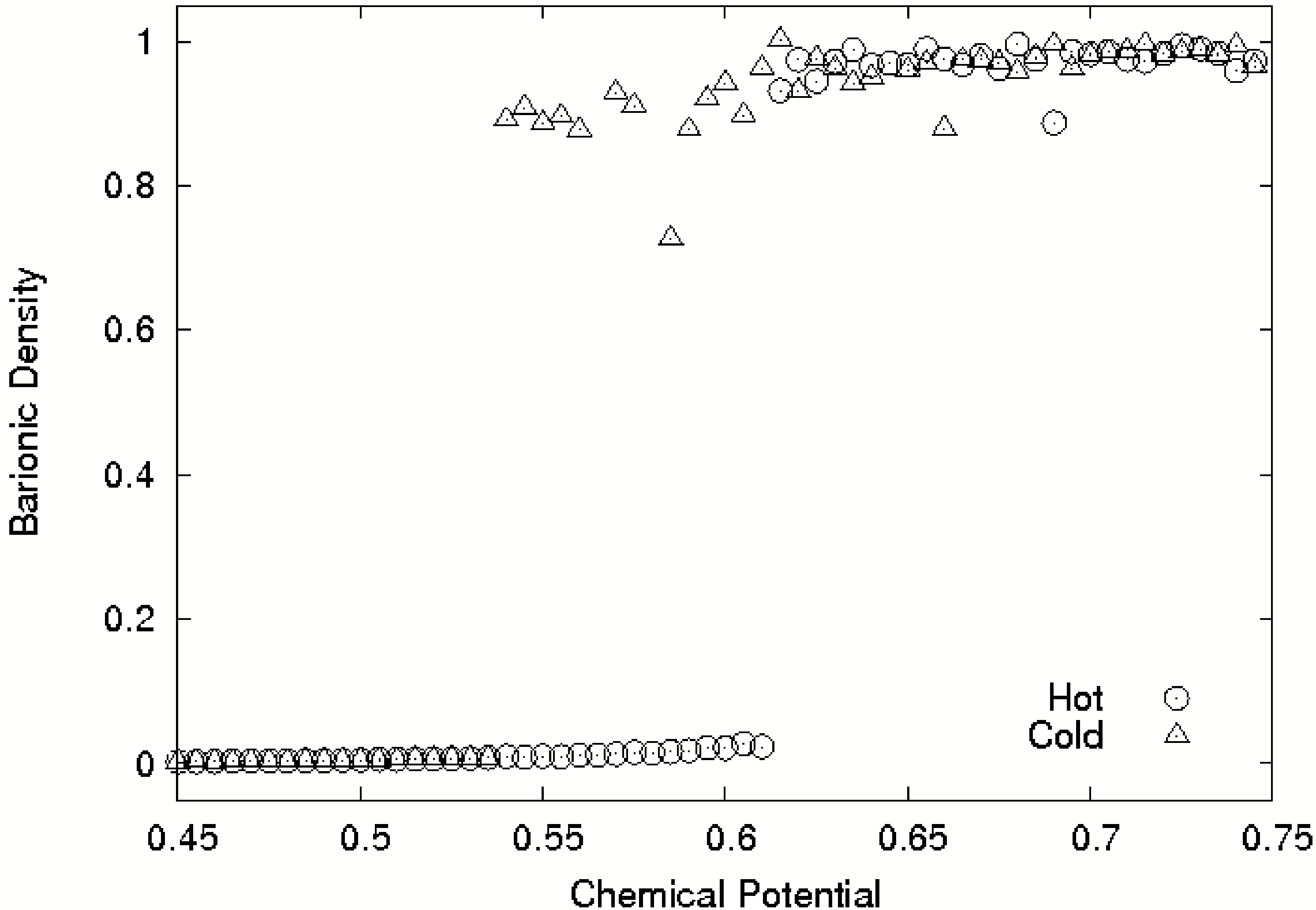
    Probability $\circ(1)$



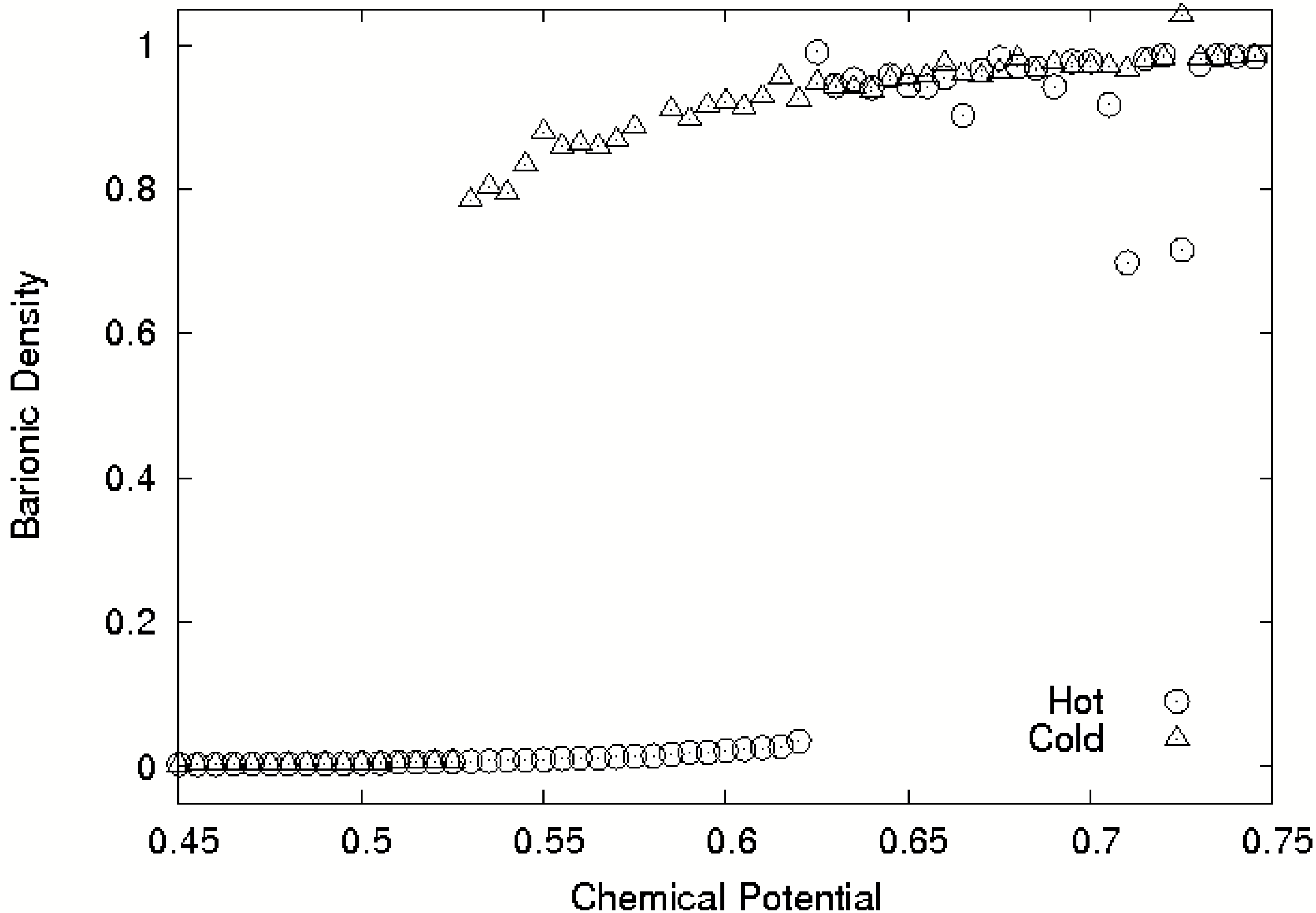
# Results: $10^3$x2, comparison with the Worm Algorithm



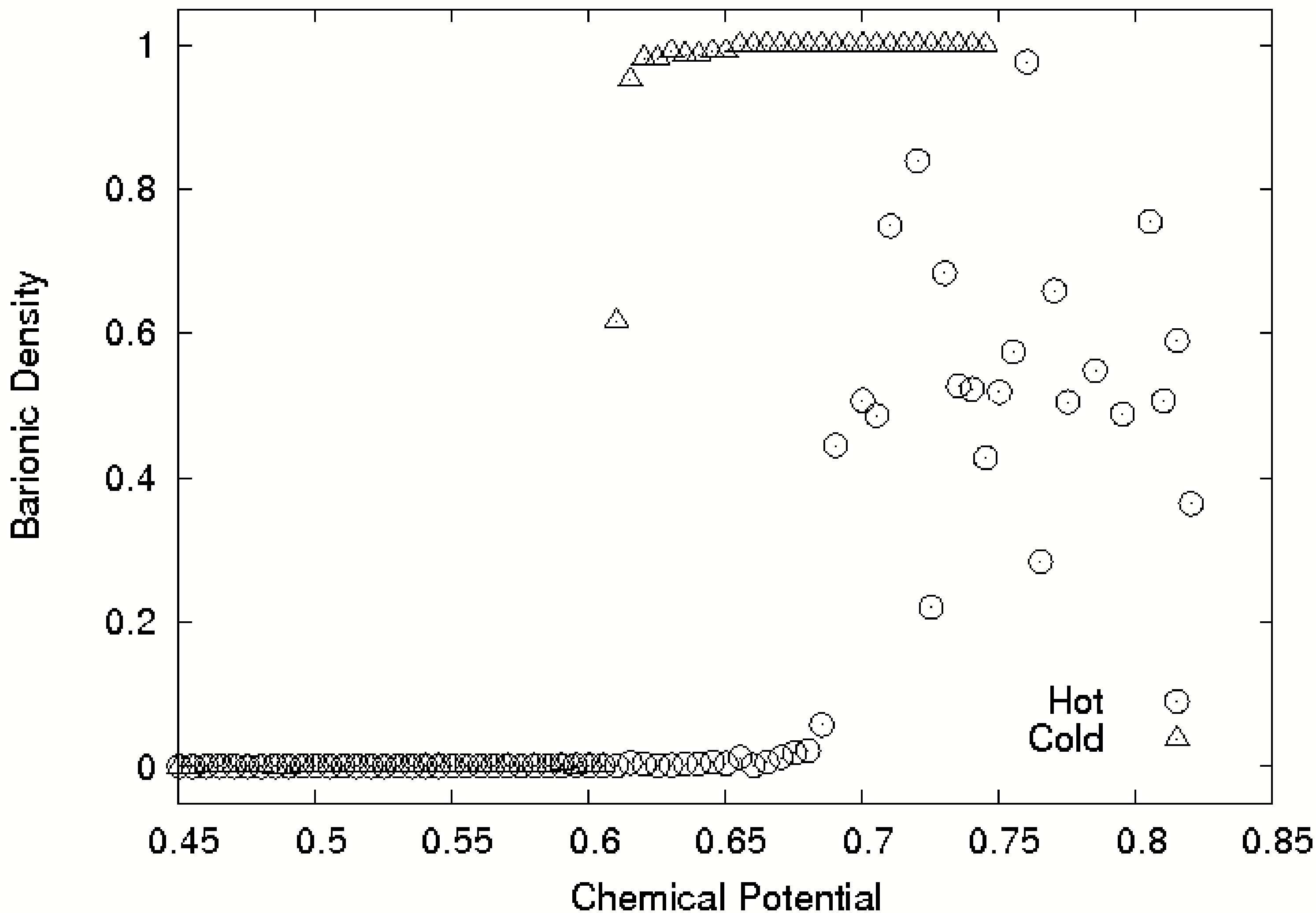
# Results: $4^3x4$



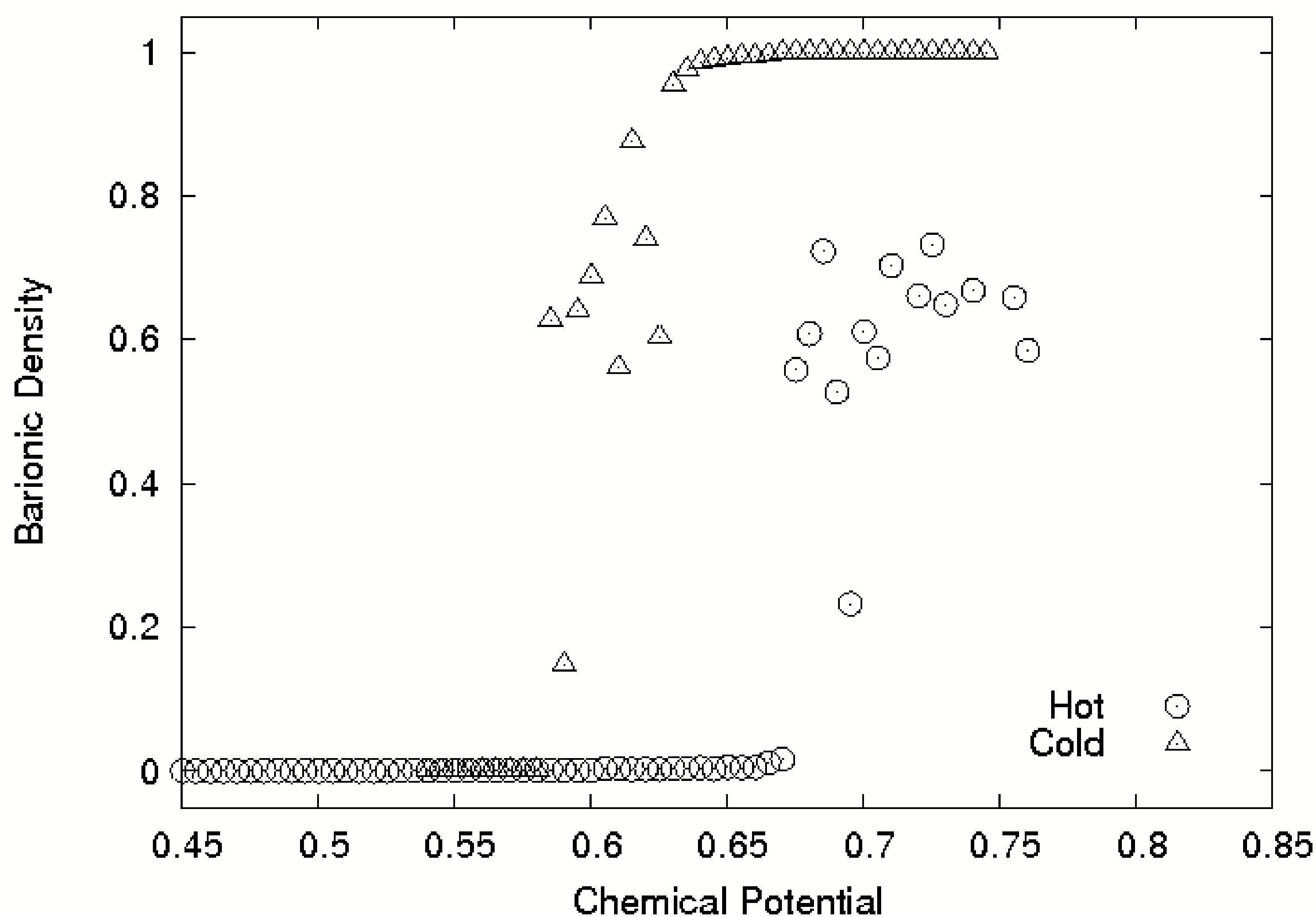
# Results : $10^3 x 4$



# Results : $6^3x6$



# Results : $10^3x6$



# Final remarks

- Ergodicity problems seems to be solved
- Be careful with the thermodynamic limit
- The sign problem is there