Compact self-graviting solutions of quartic (K) fields in brane cosmology

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[based on work in collaboration with C. Adam, N. Grandi, P. Klimas and A. Wereszczynski]

Introduction

- higher dimensional models
- methods of hiding extra dimensions
  - compactification
  - Randall – Sundrum proposal
- braneworld picture

Thick branes and localization of gravity

[C. Csaki, J. Ehrlich, T. Hollowood, Y. Shirman, NP B581 (2000) 309]

- Thick brane = solution of a higher (D) dim. gravity with 4-dim. Lorentz invariance (here $D = 5$)

$$ds^2 = e^{-A(y)} dx^2 - dy^2 = e^{-A(\tilde{y})} (dx^2 - d\tilde{y}^2)$$

$\uparrow$ warp factor $\qquad$ $\uparrow$ Minkowski line element

- Most general Em tensor with these symmetries

$$T_{AB} = \frac{1}{2} \left[\Lambda(y) g_{AB} + \mathcal{V}(y) g_{\mu\nu} \delta^{\mu}_{A} \delta^{\nu}_{B} \right]$$

$$A = (\mu, 4),\ x^A = (x^\mu, x^4 \equiv y)$$

- Einstein eqs.

$$\begin{aligned} \Lambda(y) &= -3\kappa^{-2}A_y^2 \\ \mathcal{V}(y) &= 3\kappa^{-2}A_{yy} \end{aligned}$$

$A_y \equiv \partial_y A$, etc.

- From action ($\kappa^{-2} = M_{Pl}^3$ is grav. coupling)

$$S = -\int d^5x \left[\sqrt{g_5}\left(\kappa^{-2}R_5 + \Lambda(y)\right) + \sqrt{g_4}\mathcal{V}(y)\right]$$

$g_5 \equiv |\det g_{AB}|$, $g_4 \equiv |\det g_{\mu\nu}|$

Gravitational fluctuations

$$ds^2 = e^{-A(\tilde{y})}\left[(\eta_{\mu\nu} + h_{\mu\nu}(x,\tilde{y}))dx^\mu dx^\nu - d\tilde{y}^2\right]$$

$\Rightarrow$ linear perturbation of Einstein tensor plus linear pert of Em tensor

$$\delta T_{AB} = T_A^C h_{CB}$$

in transverse traceless gauge $\partial_A h^{AB} = 0 = h_A^A$ leads to

$$-\partial_A \partial^A h_{\mu\nu} + \frac{3}{2}\partial^B A \partial_B h_{\mu\nu} = 0$$

- redefine $h_{\mu\nu}(x,\tilde{y}) = e^{\frac{3}{4}A}\tilde{h}_{\mu\nu}$
- product ansatz $\tilde{h}_{\mu\nu} = H_{\mu\nu}(x)\Psi(\tilde{y})$ then

$$\begin{aligned}\Box_4 H_{\mu\nu} &= m^2 H_{\mu\nu}\\ \left(-\partial_{\tilde{y}}^2 + U(\tilde{y})\right)\Psi &= m^2\Psi\\ U(\tilde{y}) &\equiv \frac{9}{16}A_{\tilde{y}}^2 - \frac{3}{4}A_{\tilde{y}\tilde{y}}\end{aligned}$$

- $\Rightarrow$ Schroedinger Eq. with $E = m^2 \geq 0$,

$$\left(-\partial_{\tilde{y}}^2 + U(\tilde{y})\right)\Psi = \left(-\partial_{\tilde{y}} + \frac{3}{4}A_{\tilde{y}}\right)\left(\partial_{\tilde{y}} + \frac{3}{4}A_{\tilde{y}}\right)\Psi \geq 0$$

- Zero mode

$$(\partial_{\tilde{y}} + \frac{3}{4}A_{\tilde{y}})\Psi = 0 \Rightarrow \Psi = e^{-\frac{3}{4}A}$$

- If zero mode is normalizable, $\int d\tilde{y} e^{-\frac{3}{2}A} < \infty$, then induced gravity on brane is 4-d Newtonian:

$$\int d^5x\sqrt{g_5}R_5 \sim \int d\tilde{y} e^{-\frac{3}{2}A} \int d^4x\sqrt{g_4}R_4$$

- If other modes have mass gap $m^2 > 0$, or tunneling to brane is suppressed, then corrections to 4-dim gravity are negligible $\Rightarrow$ gravity localized on brane

Example: scalar field

$$S = \int d^5x\sqrt{g_5}\left(-\kappa^{-2}R_5 + \frac{1}{2}\partial_B\phi\partial^B\phi - V(\phi)\right)$$

where $\phi = \phi(\tilde{y})$ or $\phi = \phi(y)$ is a kink solution.

- Einstein equations

$$\begin{aligned} V(\phi) - \frac{1}{2}\phi_y^2 &= -3\kappa^{-2}A_y^2 \\ \phi_y^2 &= 3\kappa^{-2}A_{yy} \end{aligned}$$

- Em tensor

$$T_{AB} = \frac{1}{2}\left[\partial_A\phi\partial_B\phi - g_{AB}\left(\frac{1}{2}g^{CD}\partial_C\phi\partial_D\phi - V(\phi)\right)\right]$$

- metric fluctuations:

$$\delta T_{AB} = T_A^C h_{CB} \text{ because } h^{AB}\partial_B\phi \sim h^{\mu\nu}\partial_\nu\phi = 0$$

- Therefore, above fluctuation analysis for the metric field applies
- BUT: scalar field fluctuations $\delta\phi$ will leave brane

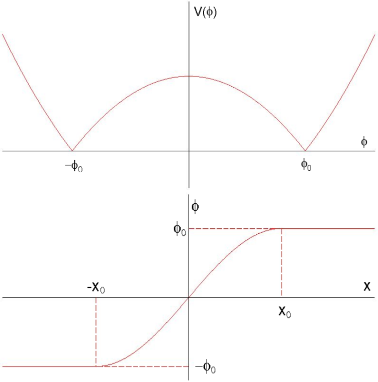
Compactons in field theory

Compacton = soliton (kink) with compact support

- V-shaped potentials

  H. Arodz, (2002), (2004)
  H. Arodz, P. Klimas, T. Tyranowski, (2005 - 2008)
  H. Arodz, J. Lis, (2008)

  - non-vanishing first derivative of the potential at the minimum
  - quadratic approach to vacuum



2. Compactons

- Compacton = soliton (kink) with compact support
- example scalar field in 1+1 dim

$$S = \int d^2x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

$$\partial_\mu\partial^\mu\phi + V' = 0$$

- static solutions

$$\phi_{xx} - V' = 0$$

$$\phi_x^2 = 2(V(\phi) - V_0)$$

where $V_0 \equiv V(\phi_{0i}) \ldots$ minimum and $\phi_{0i} \ldots$ vacuum values

- Approach to vacuum (e.g. from below): $\phi \sim \phi_0 - \delta\phi$
- For a normal U-shaped potential

$$V = V_0 + \frac{1}{2} V''(\phi_0)\delta\phi^2 + \dots$$

$$\Rightarrow \quad \delta\phi_x^2 \sim V''(\phi_0)\delta\phi^2$$

$$\Rightarrow \quad \delta\phi \sim e^{-\sqrt{V_0''}x}$$

- exponential approach to vacuum (exponential tail) . . . compactons cannot exist.
- normal topological kinks may of course exist (need at least two vacua).

- Approach to vacuum (e.g. from below): $\phi \sim \phi_0 - \delta\phi$
- For a V-shaped potential

$$V = V_0 + |V'_{0-}|\delta\phi + \dots \qquad V'_{0-} \equiv \lim_{\phi \nearrow \phi_0} V'(\phi) < 0$$

$$\Rightarrow \quad \delta\phi_x^2 \sim 2|V'_{0-}|\delta\phi$$

$$\Rightarrow \quad \delta\phi \sim \frac{1}{2}|V'_{0-}|(x - x_0)^2$$

where $V''(\phi_0)$ does not exist

- parabolic approach to vacuum a some finite $x = x_0 \dots$ compactons can exist.
- Topological compactons need at least two V-shaped vacua (W-shaped potential).

K–fields

- K–fields = fields with nonstandard kinetic terms
- Consider only $\mathcal{L}(\phi, \partial_\mu \phi)$ (no higher derivatives)
- Some uses of K–fields:
  - Evade Derrick's theorem, as in Skyrme and related models
  - K essence in cosmology: explains "coincidence" (numerical similarity of dark energy density and matter density), avoids stability problems of quintessence

    C. Armendariz-Picon, T. Damour, V. Mukhanov, (1999)
    C. Armendariz-Picon, V. Mukhanov, Paul J. Steinhardt (2000)
    J.K. Erickson, R.R. Caldwell, P.J. Steinhardt, C. Armendariz-Picon, V. F. Mukhanov (2002)
    E. Babichev, V. Mukhanov, A. Vikman (2008)
  - Here: allows for compacton solutions, restricts fluctuations to compacton region

    C. Adam, J. S-G., A. Wereszczynski, (2007), (2008)
    C. Adam, N. Grandi, P. Klimas, J. S-G. , A. Wereszczynski, (2008)

K field compacton example

- scalar field in 1+1 dim

$$S = \int d^2x(4|X|X - V(\phi)) \quad , \qquad X \equiv \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$8\partial_\mu(|X|\partial^\mu\phi) + V' = 0$$

- static solutions

$$4\partial_x(\phi_x^3) - V' = 0$$

$$3\phi_x^4 = V(\phi) - V_0$$

where $V_0 \equiv V(\phi_{0i})$ ... minimum and ϕ_{0i} ... vacuum values

- Approach to vacuum (e.g. from below): $\phi \sim \phi_0 - \delta\phi$
- For a normal U-shaped potential

$$V = V_0 + \frac{1}{2} V''(\phi_0)\delta\phi^2 + \dots$$

$$\Rightarrow \quad \delta\phi_x^2 \sim \sqrt{\frac{1}{6} V''(\phi_0)\delta\phi}$$

$$\Rightarrow \quad \delta\phi \sim \frac{1}{4}\sqrt{\frac{1}{6} V''(\phi_0)(x - x_0)^2}$$

- parabolic approach to vacuum a some finite $x = x_0$ ... compactons can exist.
- Topological compactons need at least two vacua.

Compactons and thick branes

C. Adam, N. Grandi, J. S-G. , A. Wereszczynski (2008)

- Example: scalar field in 4+1 dim

$$S = \int d^5x \left(4|X|X - 3\lambda^4(\phi^2 - a^2)^2\right) \quad , \qquad X \equiv \frac{1}{2}\partial_A\phi\partial^A\phi$$

$$8\partial_A(|X|\partial^A\phi) + 12\lambda^4\phi(\phi^2 - a^2) = 0$$

- has static one-dim compacton solution ($x^4 \equiv y$)

$$\phi_y^4 = \lambda^4(\phi^2 - a^2)^2$$

$$\phi(y) = \begin{cases} -a & y \le -\frac{\pi}{2\lambda} \\ a\sin\lambda y & -\frac{\pi}{2\lambda} \le y \le \frac{\pi}{2\lambda} \\ a & y \ge \frac{\pi}{2\lambda}, \end{cases}$$

Linear perturbation

- $\phi = \phi_c + \eta(x^\mu, y)$, $\phi_c \ldots$ compacton
- Linear perturbation equation

$$-(\partial_y \phi_c)^2_y \eta_y - (\partial_y \phi_c)^2 \eta_{yy}$$
$$+\frac{1}{3}(\partial_y \phi_c)^2 \Box_{(4)} \eta + \lambda^4 (3\phi_c^2 - a^2)\eta = 0$$

- In vacuum region $\phi_c = \pm a$, $\partial_y \phi_c = 0$

$$\lambda^4 (3\phi_c^2 - a^2)\eta = 0 \quad \Rightarrow \quad \eta = 0$$

- There are NO linear perturbations outside the compacton region

- Inside change $z = \lambda y$ and ansatz $\eta(x^A) = \bar{\eta}(z)\phi(x^\mu)$
- Equation for ϕ ordinary KG, canonical scalar field in 4-dim Minkowski space.
- Equation for $\bar{\eta}$, Schr. with positive semidefinite H operator
- Exactly one zero mode, Goldstone mode of translation $z \rightarrow z + z_0$

Backreaction and localization of gravity

- Inclusion of gravity

$$S = \int d^5x \sqrt{g_5} \left(\kappa^{-2}(R - \Lambda) + 4|X|X - V(\phi) \right)$$

where Λ is the bulk cosmological constant, and X now includes the metric

$$X = \frac{1}{2} g^{AB} \partial_A \phi \partial_B \phi.$$

$$V = 3\lambda^4(\phi^2 - a^2)^2$$

- Ansatz for metric

$$ds^2 = e^{-A(y)}\left(dt^2 - d\vec{x}^{\,2}\right) - dy^2$$

- Compacton ansatz $\phi = \phi(y)$
- $\Rightarrow$ Einstein equations

$$\frac{3}{4}A_{yy} - A_y^2 = \frac{1}{3}\left[\Lambda + \kappa^2 3\lambda^4(\phi^2 - a^2)^2\right]$$
$$\frac{3}{4}A_{yy} = \kappa^2\phi_y^4$$

- Field equation (not independent)

$$-8A_y\phi_y^3 + 12\phi_y^2\phi_{yy} = 12\lambda^4\phi(\phi^2 - a^2)$$

Is there a compacton?

- Outside R and S solution (AdS), for thick brane
- Inside proper behaviour at the boundary
- dimensionless $z = \lambda y$, $\Lambda \to \lambda^2 \Lambda$, $\lambda \to \lambda^{-1}\kappa$, $\phi \to a\phi$, $V \to a^{-4} V$, with new function $C(z) = \tilde{A}_z(z)$

Einstein equations

$$\frac{3}{4} C_z - C^2 = \frac{1}{3}\Lambda + \kappa^2 (\phi^2 - 1)^2$$

$$\frac{3}{4} C_z = \kappa^2 \phi_z^4$$

Expansion about the compacton boundary

- Power series expansion at $t = 0$ where $t = z - z_-$

$$\phi = -1 + b_2 t^2 + b_3 t^3 + b_4 t^4 + \ldots$$
$$C = -\sqrt{\frac{\bar{\Lambda}}{3}} + c_1 t + c_2 t^2 + \ldots$$

- at $z = z_-$ compacton matches vacuum solution (AdS solution of Randall and Sundrum)
- $b_2 = \frac{1}{2}$ like for case without gravity
- $b_3 = -\frac{1}{15}\sqrt{\frac{\bar{\Lambda}}{3}} < 0$ - Λ tends to increase the compacton
- b_8 – first contribution from κ (positive) - gravity tends to shrink the compacton

Compacton existence and finetuning

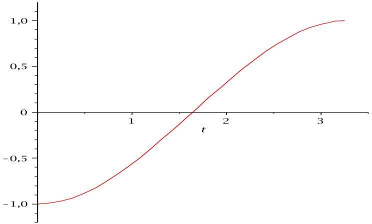
- Einstein equations for compacton case

$$
\begin{aligned}
C_z &= \frac{4}{3}\left(C^2 + \kappa^2(\phi^2 - 1)^2 - \frac{\bar{\Lambda}}{3}\right) \\
\phi_z &= \frac{1}{\sqrt{\kappa}}\left(C^2 + \kappa^2(\phi^2 - 1)^2 - \frac{\bar{\Lambda}}{3}\right)^{1/4}
\end{aligned}
$$

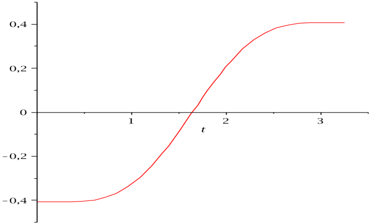
- $\phi(z = z_0) = 0$, ϕ - odd funcion about z_0
- then $C(z_0) = 0$
- solution that starts from z_- is completely determined and there is no reason that $C(z)$ reaches its zero exactly at $z = z_0 \Rightarrow$ finetuning $\bar{\Lambda}$ and κ

Numerical solution

- Solution for $\bar{\Lambda} = 0.5$ and $\kappa^2 = 0.633226$

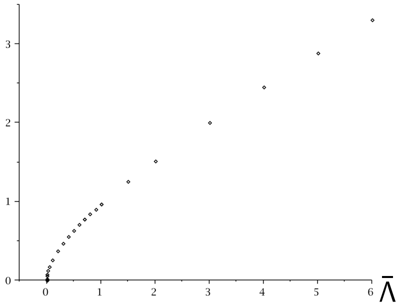


The solution $\phi(t)$

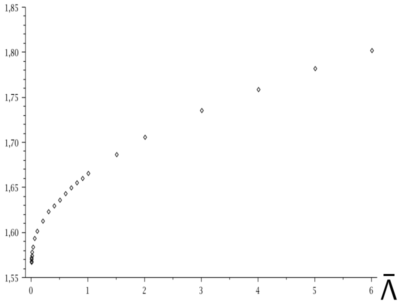


The solution $C(t)$

Finetuning and compacton radius



Relation $\kappa^2(\bar{\Lambda})$



Compacton radius $\rho(\bar{\Lambda})$

Conclusions

- Non standard kinetic term is rather strong assumption (quadratic term $(\partial_A \phi)^2$ efectively absent at low energies), but once assumed:
- Thick branes generated dynamically
- Confinement of perturbations ("particles") to brane is automatic consequence
- "matter" fields inside brane are standard, despite non-standard kinetic term

Conclusions

- With gravity included:
  - vacuum solution (outside brane) = AdS solution of R and S
  - proof of stability of the system under gravitational backreaction
  - Exact proof of linear stability
- Future work
  - adding fermions, SUSY and quantum corrections (WKB)
  - Qballs and black holes
  - possible phenomenological implications for cosmology?