Monodromy Matrix for the Black Holes

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Introduction

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- Monodromy matrix for Black Holes.

The General Construction

- In the conformal gauge $(g_{\alpha\beta} = \eta_{\alpha\beta} e^{\bar{\phi}})$, the action for a sigma model (derived from dimensional reduction of the effective string theory to two dimensions) takes the following form

$$S_\sigma = \frac{1}{8} \int dx^0 dx^1 e^{-\bar{\phi}} \eta^{\alpha\beta} Tr \left(\partial_\alpha M^{-1} \partial_\beta M \right)$$

The General Construction

- $$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix}$$

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- The moduli G and B parameterize the coset $O(d,d)/O(d) \times O(d)$ under the global $O(d,d)$ transformation,

$$M \longrightarrow \Omega^T M \Omega \qquad , \qquad \Omega \in O(d,d) \qquad (2)$$

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$$M = VV^T \qquad , \qquad V \in \frac{O(d,d)}{O(d) \times O(d)}$$

- M is a $2d \times 2d$ where V has the triangular form

$$V = \begin{pmatrix} E^{-1} & 0 \\ BE^{-1} & E^T \end{pmatrix} \tag{5}$$

The General Construction

- the with $(E^T E)_{ij} = G_{ij}$. The matrix M parameterizing the coset $\frac{O(d,d)}{O(d)\times O(d)}$ transforms nontrivially under global $O(d,d)$ as well as a local $O(d)\times O(d)$ as

$$V \longrightarrow \Omega^T V h(x) \quad , \quad \Omega \in O(d,d) \quad , \quad h(x) \in O(d)\times O(d)$$

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$$V \longrightarrow \Omega^T V h(x) \quad , \quad \Omega \in O(d,d) \quad , \quad h(x) \in O(d)\times O(d)$$

- and consequently

$$M = VV^T \longrightarrow \Omega^T V V^T \Omega = \Omega^T M \Omega$$

The General Construction

- Next, we construct the current $V^{-1}\partial_\alpha V$ which belongs to the Lie algebra of $O(d,d)$ and it can be decomposed as

$$V^{-1}\partial_\alpha V = P_\alpha + Q_\alpha \tag{6}$$

The General Construction

- Next, we construct the current $V^{-1}\partial_\alpha V$ which belongs to the Lie algebra of $O(d,d)$ and it can be decomposed as

$$V^{-1}\partial_\alpha V = P_\alpha + Q_\alpha \tag{7}$$

- Here, Q_α belongs to the Lie algebra of the maximally compact subgroup $O(d) \times O(d)$ and P_α belongs to the complement. Furthermore, it follows from the symmetric space automorphism property of the coset $\frac{O(d,d)}{O(d)\times O(d)}$ that $P_\alpha^T = P_\alpha$, $Q_\alpha^T = -Q_\alpha$;

The General Construction

- therefore,

$$P_\alpha = \frac{1}{2}\left(V^{-1}\partial_\alpha V + (V^{-1}\partial_\alpha V)^T\right) \qquad (8)$$

$$Q_\alpha = \frac{1}{2}\left(V^{-1}\partial_\alpha V - (V^{-1}\partial_\alpha V)^T\right)$$

It is now straightforward to show that

$$Tr\left(\partial_\alpha M^{-1}\partial_\beta M\right) = -4Tr\left(P_\alpha P_\beta\right) \qquad (9)$$

The General Construction

- Furthermore, the currents in (6) are invariant, under global $O(d,d)$ rotations whereas under a local $O(d) \times O(d)$ transformation, $V \longrightarrow Vh(x)$,

$$
\begin{aligned}
P_\alpha &\longrightarrow h^{-1}(x) P_\alpha h(x) \\
Q_\alpha &\longrightarrow h^{-1}(x) Q_\alpha h(x) + h^{-1}(x) \partial_\alpha h(x)
\end{aligned}
\tag{10}
$$

The General Construction

- Thus, Q_α transforms like a gauge field under local $O(d) \times O(d)$ transformations, while P_α transforms in the adjoint representation under a gauge transformation.
  ♣Let us introduce a one parameter family of matrices $V(x,t)$, t being the spectral parameter (we denote time coordinate as x^0), such that $V(x, t=0) = V(x)$ and

$$V^{-1}\partial_\alpha V = \ Q_\alpha + \frac{1+t^2}{1-t^2} P_\alpha + \frac{2t}{1-t^2}\epsilon_{\alpha\beta}P^\beta$$

The General Construction

- When we consider a sigma model in curved space-time, it is necessary for the spectral parameter to be a local function satisfying the first order differential equation

$$\partial_\alpha e^{-\bar{\phi}} = -\frac{1}{2}\epsilon_{\alpha\beta}\partial^\beta \left(e^{-\bar{\phi}}(t + \frac{1}{t}) \right)$$

The General Construction

- In order to fulfill consistency conditions arising from integrability properties. The solution for the shifted dilaton can be written as a sum

$$\rho(x) = e^{-\bar{\phi}(x)} = \rho_+(x^+) + \rho_-(x^-)$$

The General Construction

- in terms of which the solution to eq (11) is expressed as

$$t(x) = \frac{\sqrt{\omega + \rho_+} - \sqrt{\omega - \rho_-}}{\sqrt{\omega + \rho_+} + \sqrt{\omega - \rho_-}}$$

The General Construction

- It is straightforward to check that the zero curvature condition following from (10) leads to the integrability of the current in (6) as well as the dynamical equation for the nonlinear sigma model.
  ♣ In the presence of the spectral parameter, the symmetric space automorphism can be generalized as

$$\eta^{\infty}\left(\hat{V}(x,t)\right) = \eta\left(\hat{V}(x,\frac{1}{t})\right) = \left(\hat{V}^{-1}(x,\frac{1}{t})\right)^{T}$$

$$\left(\hat{V}^{-1}(x,\frac{1}{t})\partial_{\alpha}\hat{V}(x,\frac{1}{t})\right)^{T} = -\hat{V}^{-1}(x,t)\partial_{\alpha}\hat{V}(x,t)$$

The General Construction

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- It follows, from eq (14) that

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- In other words, $\mathcal{M} = \mathcal{M}(\omega)$ is independent of space-time coordinates.

Generalized two dimensional string effective action

- The action in (3) can be generalized by adding n Abelian gauge fields, with the additional action of the form (in heterotic string theory).

$$S_A = -\frac{1}{4}\int d^D x \sqrt{-G} e^{-\bar{\phi}} \left(g^{\mu\rho} g^{\upsilon\lambda} \delta_{IJ} F^I_{\mu\nu} F^J_{\rho\lambda} \right) \qquad (11)$$

where I, $J = 1, 2, ..., n$ and

$$F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu \qquad (12)$$

- This action can also be dimensionally reduced to two dimensions and the resulting effective action takes the form

$$S_A = -\frac{1}{4}\int dx^0 dx^1 \sqrt{-g}e^{-\bar{\phi}}\left(F^I_{\alpha\beta}F^{I\alpha\beta} + 2F^I_{\alpha j}F^{I\alpha j}\right) \quad (13)$$

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- where we have defined

$$
\begin{aligned}
a_i^I &= A_i^I \\
A_\alpha^{(1)I} &= G_\alpha^I \\
A_\alpha^{(3)I} &= A_\alpha^I - a_j^I A_\alpha^{(1)j} \\
F_{\alpha\beta}^{(1)i} &= \partial_\alpha A_\beta^{(1)i} - \partial_\beta A_\alpha^{(1)i} \\
F_{\alpha\beta}^{(3)I} &= \partial_\alpha A_\beta^{(3)I} - \partial_\beta A_\alpha^{(3)I} \\
F_{\alpha\beta}^I &= F_{\alpha\beta}^{(3)I} + F_{\alpha\beta}^{(1)i} a_i^I \\
F_{\alpha i}^I &= \partial_\alpha a_i^I
\end{aligned}
\qquad (15)
$$

- In the presence of the Abelian gauge fields, the field strength, H, associated with the second rank antisymmetric tensor field, B, needs to be redefined for gauge invariance as

$$
\begin{aligned}
H_{\alpha ij} &= \partial_\alpha B_{ij} + \frac{1}{2}\left(a_i^I \partial_\alpha a_j^I - a_j^I \partial_\alpha a_i^I\right) \qquad (16)\\
H_{\alpha\beta i} &= -C_{ij}\, F^{(1)j}_{\alpha\beta} + F^{(2)}_{\alpha\beta i} - a_i^I\, F^{(3)I}_{\alpha\beta}\\
H_{\alpha\beta\gamma} &= \partial_\alpha B_{\beta\gamma} - \frac{1}{2}\, A^r_\alpha\, \eta_{rs}\, F^s_{\beta\gamma} + \text{cyc.perms.}
\end{aligned}
$$

where $A^r_\alpha = (A^{(1)i}_\alpha, A^{(2)}_{\alpha i}, A^{(3)I}_\alpha)$, $F^r_{\alpha\beta} = \partial_\alpha A^r_\beta - \partial_\beta A^r_\alpha$ and

$$
\begin{aligned}
A^{(2)}_{\alpha i} &= B_{\alpha i} + B_{ij} A^{(1)j}_{\alpha} + \frac{1}{2} a^I_i A^{(3)I}_{\alpha} \\
F^{(2)}_{\alpha\beta i} &= \partial_\alpha A^{(2)}_{\beta i} - \partial_\beta A^{(2)}_{\alpha i} \\
C_{ij} &= \frac{1}{2} a^I_i a^I_j + B_{ij}
\end{aligned} \tag{17}
$$

- In two space-time dimensions, the field strength $H_{\alpha\beta\gamma}$ can be set to zero.
  Furthermore, keeping all other terms, the complete two dimensional string effective action can be shown to have the same form as in the action with

$$M = \begin{pmatrix} G^{-1} & -G^{-1}C & -G^{-1}a^T \\ -CG^{-1} & G + C^T G^{-1} C & C^T G^{-1} a^T + a^T \\ -aG^{-1} & aG^{-1}C + a & 1 + aG^{-1}a^T \end{pmatrix} \tag{18}$$

- In this case, M is a symmetric $d \times (d+n)$ matrix $(d = D-2)$ belonging to $O(d, d+n)$ and the parameter of transformation $\Omega \in O(d, d+n)$ satisfying $\Omega^T \eta \Omega = \eta$ where

$$\eta = \begin{pmatrix} 0 & 1_d & 0 \\ 1_d & 0 & 0 \\ 0 & 0 & 1_d \end{pmatrix} \tag{19}$$

- represents the metric for $O(d, d+n)$. the matrix V in this case has the following form,

$$V = \begin{pmatrix} E^{-1T} & 0 & 0 \\ -C^T E^{-1T} & E^T & a^T \\ -aE^{-1T} & 0 & 1 \end{pmatrix} \tag{20}$$

Black Holes Application

- We will discuss the black hole solutions in string theory context, particularly, in heterotic string theory, the 10-dimensional string effective action is written as

$$S = \int d^{10}x\sqrt{-G}e^{-\bar{\phi}}\left[R + (\partial\phi)^2 - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{1}{4}\delta^{IJ}F^I_{\mu\nu}F^{\mu}\right. \quad (21)$$

with $I = 1, 2, ..., 16$

- The black hole solutions, in four dimensions, are described in terms of the Einstein metric. The reduction of (21) to four dimensions, in the Einstein frame, is carried out by identifying

$$G_{\mu\nu} = \begin{pmatrix} e^{2\bar{\phi}} g_{\mu\nu} + G_{ij} A_\mu^{(1)i} A_\nu^{(1)j} & A_\mu^{(1)i} G_{ij} \\ A_\nu^{(1)j} G_{ij} & G_{ij} \end{pmatrix} \quad (22)$$

where $\mu = \tau, r, \theta, \phi$ and $i, j = 4, 5, ..., 10$.

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where $\mu = \tau, r, \theta, \phi$ and $i, j = 4, 5, ..., 10$.

- This leads to the four dimensional action of the form

$$S_4 = \int d\tau d^3x\sqrt{-g}\begin{pmatrix} R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12}e^{-2\bar{\phi}}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \\ e^{-\bar{\phi}}F^i_{\mu\nu}M^{-1}F^{\mu\nu i} + \frac{1}{8}Tr(\partial_\mu M^{-1}\partial^\mu M) \end{pmatrix} \tag{24}$$

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- In this case $M \in O(6,22)$ and the moduli parameterize the coset $\frac{O(6,22)}{O(6)\times O(22)}$.

- We are interested in charged, non-rotating, spherically symmetric black hole solutions which are described by a general metric of the form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\lambda(r)d\tau^2 + \lambda^{-1}(r)dr^2 + R^2(r)(d\theta^2 + \sin^2\theta d\phi \tag{26}$$

- Furthermore, the Maxwell equation, together with the Bianchi identity, determines that the only non-zero components of the field strengths have the forms (as 28-dimensional column matrices)

$$F_{\tau r} = \frac{\lambda(r)}{r^2} e^{\phi} M \alpha \quad , \quad F_{\theta\phi} = \sin\theta \eta \beta \tag{27}$$

where α, β are 28 component column vectors representing the electric and the magnetic charges and η is the metric of $O(6, 22)$.

- We will dimensionally reduce the effective action to three dimensions first. The metric is parameterized by

$$G_{\mu\nu} = \begin{pmatrix} e^{2\bar{\phi}} h_{\alpha\beta} + G_{mn} A_\alpha^{(1)m} A_\beta^{(1)n} & A_\alpha^{(1)m} G_{mn} \\ A_\beta^{(1)n} G_{mn} & G_{mn} \end{pmatrix} \quad (28)$$

where $\alpha, \beta = 1, 2, 3$ and $m, n = 0, 4, 5, ..., 10$. Here $\bar{\phi} = \phi - \frac{1}{2} \log \det G_{mn}$ is the shifted dilaton and the metric $h_{\alpha\beta}$, is in the Einstein frame (since the dilaton term has been factored out explicitly) with Euclidean signature.

- The dimensionally reduced effective action can be determined and it has the form

$$S_3 = \int d^3x\sqrt{h}\left(\begin{array}{c} R_h - (\partial\bar{\phi})^2 - \frac{1}{12}e^{-4\bar{\phi}}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma} - \\ e^{-2\bar{\phi}}F^T_{\alpha\beta}(\eta M\eta)F^{\alpha\beta} + \frac{1}{8}Tr(\partial_\alpha M^{-1}\partial^\alpha M) \end{array}\right) \tag{29}$$

where η is the metric of $O(7,23)$

$$\eta = \begin{pmatrix} 0 & I_7 & 0 \\ I_7 & 0 & 0 \\ 0 & 0 & I_{16} \end{pmatrix} \tag{30}$$

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where η is the metric of $O(7,23)$

$$\eta = \begin{pmatrix} 0 & I_7 & 0 \\ I_7 & 0 & 0 \\ 0 & 0 & I_{16} \end{pmatrix} \tag{32}$$

and the matrix, $M \in O(7,23)$, has the form given in eq (18). $F_{\alpha\beta}$ correspondingly represents a 30 component column matrix.

- The equations of motion for the gauge fields, following from the action, are

$$\partial_\alpha \left(e^{-2\bar{\phi}} \sqrt{h} (\eta M \eta) F^{\alpha\beta} \right) = 0 \tag{33}$$

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- In three dimensions, the solution of this can be represented through a duality relation as

$$e^{-2\bar{\phi}}\sqrt{h}(\eta M\eta)F^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma}\partial_\gamma\chi \qquad (35)$$

where χ represents 30 scalar fields (in a column matrix representation). Furthermore, the Bianchi identity

$$\epsilon^{\alpha\beta\gamma}\partial_\alpha F_{\beta\gamma} = 0 \qquad (36)$$

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$$\epsilon^{\alpha\beta\gamma}\partial_\alpha F_{\beta\gamma} = 0 \qquad (38)$$

- can now be written in terms of the 30 scalar fields as

$$D_\alpha \left(e^{2\bar{\phi}} (\eta M \eta) \partial^\alpha \chi \right) = 0 \tag{39}$$

The important point of this analysis is that, in 3-dimensions, the gauge fields can be traded in for scalars, which can, in principle, enlarge the coset parameterized by the moduli.

- In fact, let us define a 32×32 matrix as

$$\bar{M} = \begin{pmatrix} M - e^{-2\bar{\phi}}\chi\chi^T & e^{2\bar{\phi}}\chi & M\eta\chi - \frac{1}{2}e^{2\bar{\phi}}(\chi^T\eta\chi)\chi \\ e^{2\bar{\phi}}\chi^T & -e^{2\bar{\phi}} & \frac{1}{2}e^{2\bar{\phi}}\chi^T\eta\chi \\ \begin{matrix}\chi^T\eta M - \\ \frac{1}{2}e^{2\bar{\phi}}(\chi^T\eta\chi)\chi^T\end{matrix} & \frac{1}{2}e^{2\bar{\phi}}\chi^T\eta\chi & \begin{matrix}-e^{-2\bar{\phi}} + \\ \chi^T(\eta M\eta)\chi - \frac{1}{4}e^{2\bar{\phi}}(\chi^T\eta\chi)^2\end{matrix} \end{pmatrix}$$

- This is manifestly symmetric and satisfies

$$\bar{M}\bar{\eta}\bar{M} = \bar{\eta} \tag{40}$$

where

$$\bar{\eta} = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{41}$$

corresponds to the metric for $O(8, 24)$.

Therefore, the symmetric matrix $\bar{M} \in O(8,24)$. It is straightforward to verify that the action in (29) can be rewritten as

$$S = \int d^3x\sqrt{h}\left(R_h + \frac{1}{8}Tr(\partial_\alpha M^{-1}\partial^\alpha M)\right) \qquad (42)$$

and is invariant under the $O(8,24)$ transformations

$$h_{\alpha\beta} \longrightarrow h_{\alpha\beta} \quad , \quad \bar{M} \longrightarrow \Omega^T \bar{M}\Omega \quad , \quad \Omega^T \bar{\eta}\Omega = \bar{\eta} \qquad (43)$$

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and is invariant under the $O(8,24)$ transformations

$$h_{\alpha\beta} \longrightarrow h_{\alpha\beta} \quad , \quad \bar{M} \longrightarrow \Omega^T \bar{M}\Omega \quad , \quad \Omega^T \bar{\eta}\Omega = \bar{\eta} \qquad (45)$$

Thus, we note that, in three dimensions, the action is a sum of the Einstein Hilbert action and a nonlinear sigma model coupled to gravity defined over $\frac{O(8,24)}{(8)\times O(24)}$.

The three dimensional metric corresponding to the black hole solution of eq (26) has the form

$$ds^2 = h_{\alpha\beta}dx^\alpha dx^\beta = dr^2 + \tilde{R}^2(r)(d\theta^2 + \sin^2\theta d\phi^2) \qquad (46)$$

where $\tilde{R}(r) = \lambda(r)R(r)$.

Consequently, we can integrate out $\bar{\phi}$ in (42) to obtain

$$S = \int d^2\xi\sqrt{\gamma^{(2)}}\left(R_\gamma + \frac{1}{8}\gamma^{ab}Tr(\partial_a M^{-1}\partial_b M)\right) \qquad (47)$$

- where ξ^1, ξ^2 denote respectively r, θ and the two dimensional metric has the form

$$\gamma_{ab} = \begin{pmatrix} \tilde{R}(r) & 0 \\ 0 & \tilde{R}(r)\sin\theta \end{pmatrix} \tag{48}$$

This gives the effective two dimensional action in the context of black hole solutions and our general analysis can now be applied.

Recapitulation

- ♠The charged black hole solutions of heterotic string theory are described by the moduli and the gauge field configurations.
  ♠The gauge potentials have appropriate asymptotic behavior in order to define the associated charges.
  ♠The charged black hole can be solutions can be obtained by applying the solution generating techniques (type IIB theory).

Explicit Construction of the Monodromy Matrix for Black Hole

- For the simplest of black holes, namely, Schwarzschild and in the $B = 0$ case, we can write inside the trapped region

$$V(x) = diag(\lambda_1^{-1}, \lambda_2^{-1}, ..., \lambda_{32}^{-1})$$

$$\begin{aligned} V(x) &= diag(\lambda_1^{-1}, \lambda_2^{-1}, ..., \lambda_{32}^{-1}) \\ &= (\sqrt{-\frac{r}{r-m}}, 1, ..., 1, \sqrt{-\frac{r-m}{r}}, 1, \\ &\quad ..., 1, \sqrt{-\frac{r}{r-m}}, \sqrt{-\frac{r-m}{r}}) \end{aligned} \quad (49)$$

- Correspondingly, the M matrix has the form

$$\bar{M} = VV^T = diag\left(\begin{array}{c} -\frac{r}{r-m}, 1, ..., 1, \\ -\frac{r-m}{r}, 1, ..., 1, -\frac{r}{r-m}, -\frac{r-m}{r} \end{array}\right) \tag{50}$$

Here, we choose the moduli such that it goes over to the $O(8,24)$ metric in the asymptotic limit.

- In this case, we can obtain, in a straightforward manner

$$\begin{aligned} Q_\alpha &= 0 \\ P_\alpha &= diag(-\lambda_1^{-1}\partial_\alpha\lambda_1, 0, ..., 0, -\lambda_8^{-1}\partial_\alpha\lambda_8, \\ &\quad 0, ..., 0, -\lambda_{31}^{-1}\partial_\alpha\lambda_{31}, -\lambda_{32}^{-1}\partial_\alpha\lambda_{32}) \end{aligned}$$

- The one parameter family of potentials, in this case, satisfy

$$\hat{V}^{-1}(x,t)\partial_{\pm}\hat{V}(x,t) = \frac{1 \mp t}{1 \pm t} P_{\pm} \tag{51}$$

and can be determined to have the form

$$\hat{V}(x,t) = diag(\bar{V}_1, 1, ..., 1, \bar{V}_8, 1, ..., 1, \bar{V}_{31}, \bar{V}_{32}) \tag{52}$$

where $(i = 1, 8, 31, 32)$

$$\bar{V}_i = \frac{t_{d+i}}{t_i} \frac{t - t_i}{t - t_{d+i}} \lambda_i = \sqrt{-\frac{t_{d+i}}{t_i} \frac{t - t_i}{t - t_{d+i}}} \tag{53}$$

- Here, we have made the identification as before

$$-\frac{t_i}{t_{d+i}} = \lambda_i^{-2} \tag{54}$$

The monodromy matrix, in this case, follows to be

$$\hat{\mathcal{M}}(\omega) = diag(\hat{\mathcal{M}}_1(\omega), 1, ..., 1, \hat{\mathcal{M}}_8(\omega), 1, ..., 1, \hat{\mathcal{M}}_{31}(\omega), \hat{\mathcal{M}}_{32}(\omega) \tag{55}$$

with $(i = 1, 8, 31, 32)$

$$\hat{\mathcal{M}}_i(\omega) = \frac{\omega_i - \omega}{\omega_i + \omega} \tag{56}$$

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$$\hat{\mathcal{M}}(\omega) = diag(\hat{\mathcal{M}}_1(\omega), 1, ..., 1, \hat{\mathcal{M}}_8(\omega), 1, ..., 1, \hat{\mathcal{M}}_{31}(\omega), \hat{\mathcal{M}}_{32}(\omega) \tag{58}$$

with $(i = 1, 8, 31, 32)$

$$\hat{\mathcal{M}}_i(\omega) = \frac{\omega_i - \omega}{\omega_i + \omega} \tag{59}$$