Electric and Magnetic Black Holes in Arbitrary Dimension

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Electric set-up

Perturbative solutions of Hilbert-Einstein-Maxwell action

$$S = \int dx^{d+1}\sqrt{-g}\left(\frac{1}{16\pi}(R - 2\lambda) + \frac{1}{4}F^2\right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

A abelian

Magnetic set-up (I)

Non-perturbative solutions of Hilbert-Einstein-YM action

$$S = \int dx^{d+1}\sqrt{-g}\,\left[\frac{1}{16\pi}\,(R-2\Lambda) - \frac{1}{2\gamma^2}\,Tr|F|^2\right]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e f^a_{bc} A^b_\mu A^c_\nu$$

A nonabelian

Magnetic set-up (II)

The magnetic black holes we treat are **Yang-type monopoles**:

Static solutions of a YM theory on $\mathbb{R}^{2k+1,1}\backslash\{0\}$ with gauge group $G = SO(2k)$, $(d-1=2k)$, which fulfill:

1. Spherically symmetric configurations

2. Nontrivial topology

Magnetic set-up (III)

- K=1 ⟶ $G = SO(2) \approx U(1)$, $d = 3$ } Dirac monopole

- K=2 ⟶ $G = SO(4) \approx \frac{SU(2)\times SU(2)}{Z_2}$, $d = 5$ } Extended-Yang monopole

- K=3,4,... ⟶ $G = SO(2k)$, $d = 2k+1$ } $SO(2k)$-monopoles

Yang monopoles are *not* Yang-type

Spherical symmetry (I)

Both magnetic and electric objects are point-like in $d+1$ dimensions ⇒ Spherically symmetric

Spherical ansatz:

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}dr^2 + r^2 d\Omega_{d-1}^2$$

Plugging into Einstein equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu} - g_{\mu\nu}\Lambda$$

Leads to…

Spherical symmetry (II)

$$e^{-2\beta} = 1 - \frac{2m(r,t)}{r^{d-2}},$$

$$e^{2\alpha} = \left[1 - \frac{2m(t,r)}{r^{d-2}}\right] \exp\left\{f(t) + \frac{16\pi}{d-1}\int_0^r \left(\frac{T_r^r - T_t^t}{r_1^{d-2} - 2m}\right) r_1^{d-1} dr_1\right\}$$

$$m(r,t) = \frac{8\pi}{d-1}\int_0^r r_1^{d-1} T_t^t dr_1$$

Spherical symmetry (III)

For our cases $m(t,r) = m(r)$ and we will call $e^{2\alpha} \equiv \Delta$

The electric configuration is consistent with the geometry

$$\Delta(r) = 1 - \frac{2m}{r^{d-2}} + \frac{Q^2}{r^{2(d-2)}} - \frac{r^2}{R^2} \qquad R = \sqrt{\frac{k(2k+1)}{\Lambda}}$$

And the magnetic one with

$$\Delta(r) = 1 - \frac{2m}{r^{2k-1}} - \frac{\mu^2}{r^2} - \frac{r^2}{R^2} \qquad (d-1=2k)$$

Three horizons at the most

Roots of $\Delta(r)$

$$\widetilde{\Delta} \equiv r^{2k-1}\Delta = -\frac{r^{2k+1}}{R^2} + r^{2k-1} - \mu^2 r^{2k-3} - 2m$$

Derivating and equating to zero

$$-\frac{1}{R^2}(2k+1)r^4 + (2k-1)r^2 - (2k-3)\mu^2 = 0$$

Two positive (and two negative) solutions at the most

No more than 3 root for $\Lambda(r)$

Region for nonextreme black holes (I)

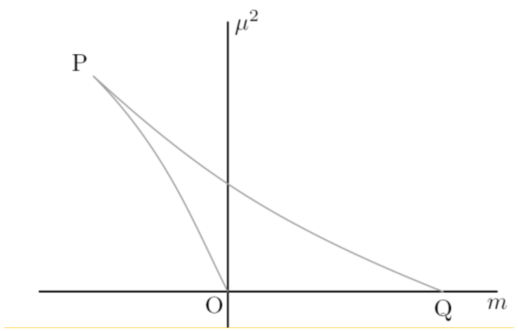
$$\Delta(r) = (r-\rho)^2 \frac{1}{r^2}\left[1 - \frac{1}{R^2}\left[r^2 + h(r)\right]\right]$$

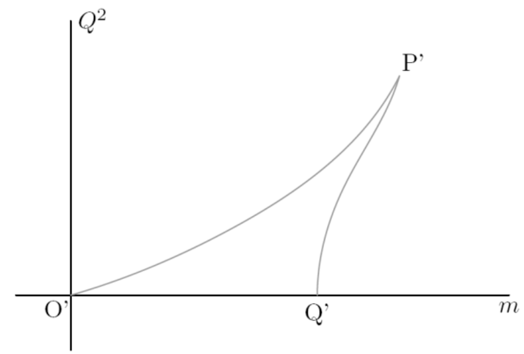
$$h(r) = a + br + \frac{c_1}{r} + \frac{c_2}{r^2} + \cdots + \frac{c_{d-4}}{r^{d-4}}$$

$$h(r) = 3\rho^2 + 2\rho r + \frac{4\rho^3 - 2\rho R^2}{r} + \frac{\mu^2 R^2 + 5\rho^4 - 3\rho^2 R^2}{r^2} + \sum_{i=1}^{d-6} \frac{(i)}{\rho^{i+1} r^{d-(i+3)}} m R^2$$

$$\begin{aligned} \mu^2(\rho) &= \frac{\rho^2}{d-4}\left[d - 2 - d\frac{\rho^2}{R^2}\right] \\ m(\rho) &= \frac{2}{d-4}\rho^{d-2}\left[2\frac{\rho^2}{R^2} - 1\right] \end{aligned}$$

Region for nonextreme black holes (II)





$$\left.\begin{aligned}\lambda_e &= \frac{Q^2}{m} R^{d-2} \\ \lambda_m &= \frac{\mu^2}{m} R^{d-4}\end{aligned}\right] \Longrightarrow \lambda^{Nariai} \leq \lambda^{Cold}$$

P and P' represent the ultracold black hole:

$$1 - \frac{1}{R^2}\left[\rho^2 + h(\rho)\right] = 0$$

Nonzero Physical distance between two coalescent horizons

If the coordinate distance between the horizons is parametrized as $r_{++} = r_+ + 2r_c\epsilon$, the physical distance

$$D(\epsilon) = \int_{r_+}^{r_{++}} \frac{dr}{\Delta^{1/2}(r)} \Longrightarrow D(\epsilon) = \int_{r_+}^{r_+ + 2r_c\epsilon} \frac{dr}{(r-r_+)^{1/2}(r_+ + 2r_c\epsilon - r)^{1/2}} \underbrace{\left(\frac{r^{k-1/2}}{g^{1/2}(r)}\right)}_{h(r)}$$

$$h_{min} \int_{r_+}^{r_+ + 2r_c\epsilon} \frac{dr}{(r-r_+)^{1/2}(r_+ + 2r_c\epsilon - r)^{1/2}} \leq D(\epsilon) \leq h_{max} \int_{r_+}^{r_+ + 2r_c\epsilon} \frac{dr}{(r-r_+)^{1/2}(r_+ + 2r_c\epsilon - r)^{1/2}}$$

$$\int_{r_+}^{r_+ + 2r_c\epsilon} \frac{dr}{(r-r_+)^{1/2}(r_+ + 2r_c\epsilon - r)^{1/2}} = \pi \Longrightarrow D(\epsilon \to 0) = \pi h(r_c)$$

Horizon coalescence geometry: GNS

$$\Delta(r) = -A^N(r)\big(r-\rho(1+\epsilon)\big)\big(r-\rho(1-\epsilon)\big)$$

with

$$A^N(r) = -\frac{1}{r^2}\left[1-\frac{1}{R^2}(r^2+h(r))\right]$$

New coordinates
$$\begin{cases} t &= \dfrac{\tau}{\epsilon A(\rho)} \\ r &= \rho(1+\epsilon\cos\chi) \end{cases}$$

$$ds_N^2 = \frac{1}{A^N(\rho)}(-\sin^2\chi d\tau^2 + d\chi^2) + \frac{1}{\rho^2}d\Omega_{d-1}^2 \quad \Longrightarrow \quad dS_2 \times S^{d-1}$$

$$\begin{aligned} A_e^N(\rho) &= (d-2)^2\frac{1}{\rho^2} - d(d-1)\frac{1}{R^2} \\ A_m^N(\rho) &= (d-2)\frac{1}{\rho^2} - 2d\frac{1}{R^2} \end{aligned}$$

Horizon coalescence geometry: GCS

$$\Delta(r) = A^{Cold}(r)\big(r-\rho(1+\epsilon)\big)\big(r-\rho(1-\epsilon)\big)$$

New coordinates
$$\begin{cases} t &= \dfrac{\tau}{\epsilon A^{Cold}(\rho)} \\ r &= \rho(1+\epsilon\cosh\chi) \end{cases}$$

$$\boxed{ds^2_{Cold} = \frac{1}{A^{Cold}(\rho)}(-\sinh^2\chi d\tau^2 + d\chi^2) + \frac{1}{\rho^2}d\Omega^2_{d-1}} \Longrightarrow AdS_2 \times S^{d-1}$$

Horizon coalescence geometry: US

New coordinates

$$\begin{cases} t_{e,m} &= \dfrac{\tau}{a_{e,m}\epsilon^{3/2}} \\ r_{e,m} &= \rho_{ce,cm}(1+\epsilon\cos\sqrt{b_{e,m}\epsilon^{1/2}}\chi) \end{cases}$$

$$a_e = (d-2)\sqrt{\frac{2}{3}} \qquad b_e = \frac{4}{R}\sqrt{\frac{d(d-1)}{3}}$$

$$a_m = \sqrt{\frac{2(d-2)}{3}} \qquad b_m = \frac{4}{R}\sqrt{\frac{d}{3}}$$

$$\begin{aligned} ds_e^2 &= -d\tau^2 + d\chi^2 + \rho_{ec}^2 d\Omega_{d-1}^2 \\ ds_m^2 &= -d\tau^2 + d\chi^2 + \rho_{mc}^2 d\Omega_{d-1}^2 \end{aligned} \quad \Longrightarrow \quad \mathcal{M}^{1,1} \times S^{d-1}$$

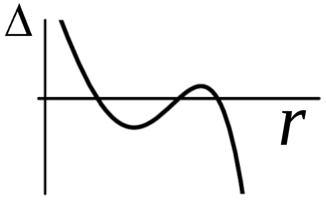
$$\rho_{ec} = \frac{d-2}{\sqrt{d(d-1)}}R \qquad \rho_{mc} = \sqrt{\frac{d-2}{2d}}R$$

Conclusions

Two different systems have been studied…

Complete analogy for the nonextremal black hole regions

Very similar horizon coalescence solutions



The process of coalescence had cleared out the differences, now encoded only in the relation between radii of the product geometries

¡*Bon appétit*!