

Extremal Black Branes on The Elliptic Curve

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June 12, 2009

Based on
Work (in preparation) with Ahl Laamara, Asorey, Segui

Motivations

- Attractor mechanism of black holes in four dimensions
Ferrara et al (1996-2009)
- Black object attractors on the K3 surface
AB. Drissi, Saidi, Segui, Nucl.Phys.B796:521-580,2008

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Plan

- 1 Generalities on Calabi-Yau Spaces
- 2 Extremal Black objects in 6D
- 3 Extremal Black objects in 8D
- 4 Conclusion

Calabi-Yau n -folds

Definition

Complex, Kahler, and existence of a global nonvanishing holomorphic n -form. Equivalently, it is a Kahler manifold with a vanishing first Chern class $c_1 = 0$ ($SU(n)$ Holonomolmy group). After compactification, it preserves only $\frac{1}{2^{n-1}}$ of of the initial supercharges.

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The moduli space contains:

$$\mathcal{M}_K \times \mathcal{M}_C$$

- \mathcal{M}_K = Kahler Moduli space : $h^{1,1}$
- \mathcal{M}_C = Complex structure Moduli space: $h^{n-1,1}$

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Hodge diagrams of Calabi-Yau $n = 1$ -folds

The case $n = 1$: The Torus T^2

$$\begin{array}{cccc}
 & h^{0,0} & & 1 \\
 h^{1,0} & & h^{0,1} = 1 & 1 \\
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 \end{array}$$

The case $n = 2$: the K3 surface

$$\begin{array}{ccccccc}
 & & h^{0,0} & & & 1 & \\
 & h^{1,0} & & h^{0,1} & & 0 & 0 \\
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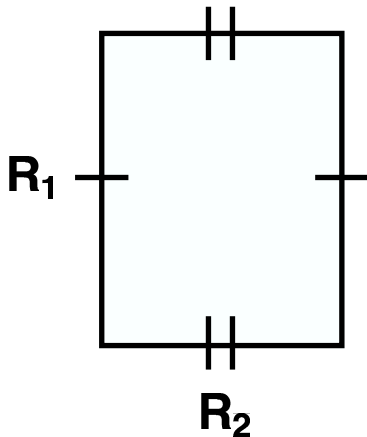
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Example: The 2-torus

The torus $T^2 = S^1 \times S^1$. One can picture it as a rectangle which one has glued together at the boundaries



T^2 has an obvious flat metric depending on two parameters R^1 , R^2 .

The moduli space of T^2 : One complex parameter (the shape) and one real parameter (the size)

- The size $\sim R^1 \times R^2 \rightarrow$ Kahler Moduli space : $h^{1,1} = 1$
- The shape $\sim i \frac{R^1}{R^2} \rightarrow$ Complex structure Moduli space:
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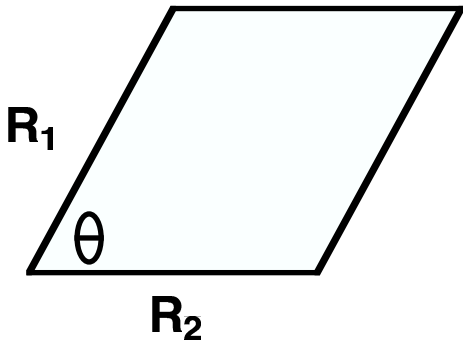
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On Extremal black branes in 6D

AB. Drissi, Saidi, Segui, Nucl.Phys.B796:521-580,2008

Consider Type IIA superstring in ten dimensions

NS-NS : g_{MN} , B_{MN} , ϕ **R-R** : A_M , C_{MNK} , $M, N, K = 0, \dots, 9$

The compactification on the K3 surface

- gives $N = (1, 1)$ supergravity in 6D
- The spectrum
 - Supergravity Multiplet
 - 20 Vector Multiplets

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The moduli space:

$$\frac{SO(4, 20)}{SO(4) \times SO(20)} \times SO(1, 1)$$

- $\frac{SO(4, 20)}{SO(4) \times SO(20)}$: the geometric deformations of the K3 surface in the presence of the antisymmetric B-field of the **NS-NS** sector
- $SO(1, 1)$: corresponds to the dilaton scalar field.

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Relation with 6D Extremal Black branes

The electric/magnetic duality

$$p + q = 2$$

This can be solved by

$p = 0$	black hole with n.h.g	$AdS_2 \times S^4 : \frac{SO(4,20)}{SO(4) \times SO(20)}$
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On The Moduli Space of $N = 2$ Supergravity in Eight Dimensions

The field content of this model consists of a

- Graviton
- Seven scalars
- Six vector fields
- Three 2-form gauge fields
- One self dual 3-form gauge field

M-theory picture (The compactification on T^3)

$$\frac{SL(3,R)}{SO(3)} \times \frac{SL(2,R)}{SO(2)}$$

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- The electric/magnetic duality

$$p + q = 4,$$

- Three different solutions

$p = 0$ ($q = 4$)	Black holes (dual black 4-branes)
$p = 1$ ($q = 3$)	Black strings (dual black 3-branes)
$p = 2$ ($q = 2$)	Dyonic black 2-branes

- The near-horizon geometries

$$AdS_{p+2} \times S^{6-p}.$$

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New Realization of The moduli space

In Type IIA superstring, the scalar manifold should take the form!

$$M_1 \times M_2 \times M_3$$

Conjecture on The CY n-folds

AB, African Journal of Mathematical Physics, Volume
6(2008)49-54

- The BH charges fix only the geometric deformations of the CY space including the **NS-NS** B-field

$p = 0$ Black hole charges fix \rightarrow geometric moduli

- The dilaton could be fixed by the dyonic object.

$p = 3 - n$ Dyonic black brane charges fix \rightarrow the dilaton

- The moduli coming from **R-R** gauge fields on CY cycles should be fixed by the higher-dimensional black object charges, like strings and branes.

$p \neq 0, 3 - n$ Black brane charges fix \rightarrow **R-R** stringy moduli.

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The T-duality role in Type IIA superstring

$$M_1 = \frac{SO(2,2)}{SO(2) \times SO(2)},$$

The dilaton in eight can be represented by a non compact circle given by

$$M_2 = SO(1,1)$$

The two remaining parameters specifying the Wilson line on T^2

$$M_3 = \frac{SO(2,1)}{SO(2)}$$

The new realization

$$\frac{SO(2,2)}{SO(2) \times SO(2)} \times \frac{SO(2,1)}{SO(2)} \times SO(1,1)$$

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Scalar Manifold Factors/ Extremal Black p -brane Charges Correspondence

The total abelian gauge group is

$$U(1)_{b2b} \times U(1)_{bs}^3 \times U(1)_{bh}^6$$

We propose the following correspondence

Coset space	Black object	Gauge symmetry
$\frac{SO(2,2)}{SO(2) \times SO(2)}$	black holes (black 4-branes)	$U(1)_{bh}^4$
$\frac{SO(2,1)}{SO(2)}$	black strings (black 3-branes)	$U(1)_{bs}^3$
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Dyonic 2-brane Attractors

- The near-horizon geometry ansatz of this configuration

$$ds^2 = r_{AdS}^2 ds_{AdS_4}^2 + r_S^2 ds_{S^4}^2, \quad H_4 = p \alpha_{S^4} + e \beta_{AdS_4},$$

- The effective potential

$$V_{\text{eff}} = \frac{1}{2}(p^2 \exp(-4\phi) + e^2 \exp(4\phi)).$$

- The attractor equation

$$\frac{dV}{d\phi} = 0, \quad \frac{d^2V}{d^2\phi} > 0.$$

- The value of the dilaton at n.h.g of black 2-brane:

$$\exp(4\phi) \sim \frac{p}{e}$$

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Conclusion

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- In the $AdS_4 \times S^4$ near-horizon limit, the examination of the black 2-brane effective potential fixes the values of the dilaton in terms of the electric and magnetic charges of black 2-branes.
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